CRITERIA OF OCCURRENCE OF FREE CONVECTION IN A COMPRESSIBLE VISCOUS HEAT-CONDUCTING FLUID

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Fundamental equations of free convection in a compressible viscous heat-conducting fluid are derived. The difference between these and the equations for an incompressible fluid has been reduced to two dimensionless parameters with respect to which the transitions to limit yield the Rayleigh and the Schwarzschild criteria. The problem is solved by the Bubnov-Galerkin method. Three characteristic parameters of length are derived from the fluid parameters, and the solution (the critical temperature gradient for convection onset) is such that it is possible to indicate the criterion applicable to individual cases by comparing the height of the fluid layer with these parameters.

1. The mechanical equilibrium of a nonuniformly heated fluid in a gravitational field to which heat is added below is steady, if the temperature gradient throughout its mass is constant and does not exceed a certain critical value [1]. If this condition is not satisfied, internal motions (free convection) appear in the fluid, which tend to equalize the temperature throughout the fluid volume.

Convective motion in the fluid is stimulated by its thermal expansion, while density variation due to hydrostatic pressure and dissipative processes taking place in a fluid in motion tend to return the fluid to its initial state.

Usually the effect of one of these two factors on the conditions for convection onset is analyzed, leading to one of the two criteria, that of Rayleigh or of Schwarzschild.

If isothermal convection, i.e. the density variation related to pressure variation $\Delta p = \rho g l_0$, where l_0 is the hight of the fluid layer expressed in terms of the thermal expansion

$$\left(\frac{\partial\rho}{\partial p}\right)_T \Delta p < -\left(\frac{\partial\rho}{\partial T}\right)_p \Delta T, \text{ or } \quad \frac{\Delta T}{l_0} > \rho g \left(\frac{\partial T}{\partial p}\right)_\rho \tag{1.1}$$

can be neglected, the convection onset is determined by the Rayleigh number – a dimensionless combination of fluid parameters, viz.

$$\frac{\rho_0 c_p g}{\lambda\eta}\left(\frac{\partial\rho}{\partial T}\right)_p l_0^3 (T_1 - T_2) \begin{cases} < \gamma_0 & \text{absence of convection} \\ \geqslant \gamma_0 & \text{presence of convection} \end{cases} \tag{1.2}$$

where all parameters in the left-hand side of (1.2) have their usual meaning, and the number γ_0 for a flat layer of fluid is equal 657.5, 1707.8 and 1100.65 in the cases of a fluid with two free, or two solid boundaries, or with the lower boundary solid and the upper free, respectively [1].

The consideration of the other limit case where the viscosity and thermal conductivity are neglected, while the compressibility is taken into account, yields for the onset of convection the Schwarzschild criterion:

$$g\rho_0\left(\frac{\partial T}{\partial p}\right)_\rho \left(1 - \frac{c_v}{c_p}\right) \begin{cases} > (-\partial T/\partial z) & \text{absence of convection} \\ < (-\partial T/\partial z) & \text{presence of convection} \end{cases} \tag{1.3}$$

It would be obviously interesting to investigate the conditions for the occurrance of convection in a compressible viscous and heat conducting fluid, which in limit cases would yield relationships (1.2) and (1.3).

This problem was qualitatively analyzed in [2]. Physical concepts help in certain cases to forecast the result, in others it is necessary to resort to quantitative calculations. Thus, for example, when $\Delta T/l_0$ satisfies the lower of conditions (1.3) and condition (1.1), then, according to Schwarzschild, convection is bound to occur – compressibility cannot overcome thermal expansion, but it is not clear whether dissipation can arrest convection, since the Rayleigh method does not apply to this case.

The attempt made in [3] at a qualitative solution of this problem is unconvincing, owing to the special assumptions as regards the model and, also, because of the difficulty of comparing his results with experimental data.

We shall determine the criterion of convection onset in a compressible viscous heat-conducting fluid by variational methods, and shall limit our analysis to a plane layer of fluid.

Solution of the general problem of convection onset criterion may prove useful in investigations of certain problems of physics of the atmosphere and, also, of the behavior of a fluid close to its critical point, where there is a sharp increase of compressibility.

The hydrodynamic equations for a compressible viscous heat-conducting fluid in a gravitational field are of the form

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\,\mathbf{v} = -\frac{\nabla p}{\rho} + \mathbf{g} + \nu\Delta\mathbf{v} + \left(\frac{\nu}{3} + \xi\right)\nabla\,\mathrm{div}\,\mathbf{v}, \qquad \frac{\partial \rho}{\partial t} + \mathrm{div}\,(\rho\mathbf{v}) = 0$$

$$\rho T\left[\frac{\partial S}{\partial t} + (\mathbf{v}\nabla)\,S\right] = \lambda\Delta T + \sigma_{ik}'\frac{\partial v_i}{\partial x_k} \tag{1.4}$$

We separate the various thermodynamic parameters in (1.4)

$$\rho = \langle\rho\rangle + \rho_0 + \rho', \quad p = p_0 + p', \quad T = \langle T\rangle + T_0 + T', \quad \langle T\rangle = \mathrm{const}, \quad \langle\rho\rangle = \mathrm{const} \tag{1.5}$$

where ρ_0, T_0 and p_0 relate to the distribution of these along the height, prior to the onset of convection, in the presence of gravity and of a temperature gradient

$$\nabla p_0 = (\langle\rho\rangle + \rho_0)\,\mathbf{g}, \quad \nabla T_0 = \mathbf{A}, \ \frac{\partial \rho_0}{\partial z} = -\left(\frac{\partial\rho}{\partial p}\right)_T (\langle\rho\rangle + \rho_0)\,g - \left(\frac{\partial\rho}{\partial T}\right)_p A \tag{1.6}$$

Finally, the presence of convection ($\mathbf{v} \neq 0$ in (1.4)) leads to variation of pressure p', temperature T', and density ρ' which are related by the equation of state

$$\rho' = -(\langle\rho\rangle + \rho_0)\,\beta T' + \left(\frac{\partial\rho}{\partial p}\right)_T p', \qquad \beta = -\frac{1}{\langle\rho\rangle + \rho_0}\left(\frac{\partial\rho}{\partial T}\right)_p \tag{1.7}$$

Since in the following only the onset of convection will be considered, we shall linearize Eqs. (1.4) with respect to small p', T' and ρ'.

Substituting (1.5)–(1.7) into (1.4), we obtain for free convection in a compressible viscous heat-conducting fluid the equation (*)

$$\frac{\partial \mathbf{v}}{\partial t} = -\frac{(\nabla - \mathbf{L}_2)}{\langle\rho\rangle + \rho_0}\,p' - \mathbf{g}\beta T' + \nu\Delta\mathbf{v} + (\xi + \nu/3)\,\nabla\,\mathrm{div}\,\mathbf{v} \tag{1.8}$$

$$\left[\frac{\partial T'}{\partial t} - \left(\frac{\partial T}{\partial p}\right)_\rho \frac{\partial p'}{\partial t}\right]\beta = \mathrm{div}\,\mathbf{v} + v_z\,(L_1 - L_2), \qquad \frac{\partial T'}{\partial t} - \left(\frac{\partial T}{\partial p}\right)_S \frac{\partial p'}{\partial t} = \varkappa\Delta T' + A\,(1 - \alpha)\,v_z$$

*) Spigel and Veronis [4] had written these equations for a perfect gas, while Jeffreys [5] took compressibility into consideration in the equation of thermal conductivity only.

Here $\varkappa = \frac{\lambda}{(\langle\rho\rangle + \rho_0)\, c_p}$, $L_1 = A\beta$, $L_2 = g\left(\frac{\partial\rho}{\partial p}\right)_T$, $\alpha = \frac{L_2}{L_1}(1 - c_v/c_p)$

Since according to (1. 6) $\rho_0 = \rho_0(z)$, and correspondingly $\beta = \beta(z), \nu = \nu(z)$ and $\varkappa = \varkappa(z)$, the coefficients in the system of Eqs. (1. 8) are, generally speaking, functions of the coordinates. Moreover, the derivatives $(\partial\rho/\partial T)_p$ and $(\partial\rho/\partial p)_T$ also vary with height. These functional relationships are particularly significant in the presence of considerable inhomogeneities, such as exist in the critical region of a fluid.

In the following the coefficients in Eqs. (1. 8) will be assumed constant. It follows from Eq. (1. 6) that (for $L_2 l_0 \lesssim 1$)

$$\rho = \langle\rho\rangle + \rho_0 \approx \langle\rho\rangle\left[1 - L_2 l_0 \frac{z}{l_0} + L_1 l_0 \frac{z}{l_0}\right] \tag{1.9}$$

i. e. we neglect in (1. 8) function $\rho(z)$, and take into account only the first corrections with respect to $L_1 l_0$ and $L_2 l_0$ in the equation of free convection in an incompressible fluid.

The solution of the system of nonstationary linear equations (1. 8) depend on time according to the law $e^{-i\omega t}$. If among acceptable values of ω there is at least one such that $\mathrm{Im}\,\omega > 0$, the stationary mode is unstable, with the instability occurring at $\mathrm{Im}\,\omega = 0$. It was shown in [6] that in the case of an incompressible fluid ω is imaginary, hence the condition $\mathrm{Im}\,\omega = 0$ reduces to $\omega = 0$. In the case of a compressible fluid considered here the assumption that $\omega = 0$ means that an increasing temperature gradient produces an initially stable convection.

Eliminating pressure and horizontal velocity components from Eqs. (1 8) and assuming (by virtue of the problem unboundedness in the horizontal direction) that the dependence on horizontal coordinates is of the form $e^{i\mathbf{kr}}$, where $\mathbf{k}$ is the two-dimensional wave vector in the xy-plane, we obtain for the amplitude of vertical velocity $v_z = f(z)$ and temperature $T' \equiv \tau(z)$ at $\omega = 0$ the equations

$$\left\{D^2 + l_0 L_1 D \frac{d}{dz} + l_0^2 L_2 (L_1 - L_2) D + l_0^2 L_2 (L_2 - L_1)\left(\frac{\xi}{\nu} + \frac{1}{3}\right) k^2\right\} f = \frac{A\beta g l_0^4}{\nu\varkappa} k^2 \tau$$

$$D\tau = -(1 - \alpha) f, \qquad D \equiv \frac{d^2}{dz^2} - k^2 \tag{1.10}$$

(in terms of units: the layer height l_0 for length, l_0^{-1} for the wave vector, $\varkappa l_0^{-1}$ for velocity, and $A l_0$ for temperature)

Boundary conditions for Eqs (1. 10) depend on whether the fluid layer is bounded by solid surfaces, or its surface is free Let us assume that the two surfaces bounding the fluid are solid, then the boundary conditions for Eqs. (1. 10) are of the form (the coordinate origin is in the middle plane)

$$f = \frac{df}{dz} = \tau = 0 \qquad \text{for} \quad z = \pm 1/2 \tag{1.11}$$

Equations (1. 10) contain both criteria of convection onset – that of Schwarzschild (1. 3), and that of Rayleigh (1. 2). In fact, if in the heat conduction equation only compressibility is taken into consideration, while viscosity and thermal conductivity are neglected, the condition $\nabla S = 0$, or in accordance with (1. 8) $\alpha = 1$ (for $\omega = 0$) represents exactly the Schwarzschild criterion (1. 3)

To obtain the second limit case, that of the Rayleigh criterion, it is necessary to disregard compressibility in Eqs. (1. 8), i. e to assume $l_0 L_2 \ll 1$, and, also, the terms containing L_1 in the first two of Eqs. (1. 8), since thermal expansion is usually taken into account only in the "driving power of convection" $-g\beta T'$ in (1.8). Then for a stationary motion Eqs. (1.10) take the usual (for an incompressible fluid) form

$$D^2 f = \frac{A\beta g l_0^4}{\nu\varkappa} k^2\tau, \qquad D\tau = -f \tag{1.12}$$

with the previously defined boundary conditions (1.11).

From Eqs. (1.12) we readily obtain one sixth-order equation with the Rayleigh number Υ as its eigenvalue, and parameter k. Function $\Upsilon = \Upsilon(k)$ is defined by the boundary conditions, and its minimum value represents the criterion of instability onset with the periodicity k_0 in the horizontal plane as the minimizing factor. We thus obtain the Rayleigh criterion (1.2) for an incompressible fluid.

2. In the case of a compressible fluid the system of Eqs. (1.10) is very cumbersome, since it depends on several parameters. This not only makes an analytical solution impossible, but also presents considerable difficulties in the derivation of a numerical one. Because of this, we follow [7] and use the Bubnov-Galerkin method for deriving an approximate solution of the problem of stability of a nonuniformly heated fluid.

Equations (1.10) differ for (1.12) by two additional parameters $L_1 l_0$ and $L_2 l_0$ which characterize the thermal expansion and the compressibility of the fluid. Therefore the eigenvalue of the problem – the critical temperature gradient is now a function of three dimensionless parameters: k, $L_1 l_0$ and $L_2 l_0$, and, also, of the ratio of specific heats c_v/c_p and of the shear and dilatational viscosities ξ/ν. All of these parameters will, obviously, appear in the function $\Upsilon(k)$ we are interested in.

We approximate the vertical velocity amplitude $f(z)$ in accordance with the boundary conditions (1.11) by the following system of even orthogonal functions (*):

$$f^{(1)}_{(z)} = \alpha_1 f_1^{(1)} + \alpha_2 f_2^{(1)} + \ldots = \alpha_1(1 + \mathrm{Cos}\, 2\pi z) + \alpha_2 \; (1 + \mathrm{Cos}\, 6\;\pi z) + \ldots \tag{2.1}$$

where $\alpha_1, \alpha_2 \ldots$ are constant coefficients which are determined from a system of algebraic equations by the Bubnov-Galerkin method.

Calculations were also carried out with an approximation function of the form

$$f_z^{(2)} = \alpha_1 f_1^{(2)} + \alpha_2 f_2^{(2)} + \ldots = \alpha_1(1 - 4z^2)^2 + \alpha_2(1 - 4z^2)^2 z^2 + \ldots \tag{2.2}$$

Substituting (2.1), or (2.2) into the second of Eqs. (1.10), we obtain the temperature distribution $\tau(z)$ defined by coefficients $\alpha_1, \alpha_2 \ldots$ and the form of function $f(z)$.

To determine coefficients α_i and the eigenvalues of the problem by the Bubnov-Galerkin method it is necessary to substitute $\tau(z)$ into the first of Eqs. (1.10), multiply it by $f_i(z)$, and integrate with respect to z.

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*) From the exact solution of the problem of an incompressible fluid follows that with increasing temperature gradient even perturbations are first realized, i.e. these are precisely the ones which determine the disruption of stability. We assume that the stability disruption is determined by perturbations (2.1) and (2.3) even with respect to z, also when taking compressibility into consideration, despite of Eq. (1.10) containing the term $l_0 L_1 D d(\ldots)/dz$ which is odd with respect to z. However the structure of this term

$$\int_{-1/2}^{1/2} \varphi_1 l_0 L_1 D \frac{d}{dz} \varphi_2 dz = -\int_{-1/2}^{1/2} \varphi_2 l_0 L_1 D \frac{d}{dz} \varphi_1 dz$$

where φ_1 and φ_2 are, respectively, odd and even functions, is such that it vanishes for both even and odd trial functions, when the method of variation is used.

As the result we obtain for the coefficients α_i the homogeneous linear set of equations

$$\sum_{k=1}^{2}\alpha_i \int_{-1/2}^{1/2} f_i\,(\mathbf{L}f_k-\gamma\tau_k)\,dz=0, \qquad \gamma=\frac{\beta g l_0^4 A}{\nu\varkappa} \qquad (i=1,2) \tag{2.3}$$

Here $\mathbf{L}f$ is the left-hand side part of the first of Eqs. (1.10).

The condition of solvability of the system of Eqs. (2.3) is the vanishing of the determinant, and this provides the equation for the determination of the problem eigenvalues $\gamma_{1,2}$ (*).

Proceeding in accordance with the described general method, we obtain equation

$$\frac{A\beta g l_0^4}{\nu\varkappa}(1-\alpha)-\gamma_0(k)-\gamma_1(k)\,L_2\,(L_2-L_1)\,l_0^2=0 \tag{2.4}$$

Here for function $f_1^{(1)}$ in (2.1)

$$\gamma_0(k) = N^{-1}(8\pi^4+4\pi^2k^2+{}^3/_2k^4),\quad \gamma_1(k)={}^1/_2N^{-1}[4\pi^2+3(\xi/\nu+{}^4/_3)k^2]$$

$$N=1-\left(\frac{4\pi^2}{k^2+4\pi^2}\right)^2\frac{\mathrm{th}\,{}^1/_2k}{{}^1/_2k}+\frac{1}{2}\frac{k^2}{k^2+4\pi^2}$$

and for function $f_1^{(2)}$ in (2.2)

$$\gamma_0(k) = E^{-1}k^9(k^4+24k^2+504),\quad \gamma_1(k)=E^{-1}k^9\,[12+(\xi/\nu+{}^4/_3)k^2]$$

$$E=k^5\,(k^4-12k^2+504)+5040\quad(12+k^2)[6k-(12+k^2)\quad \mathrm{th}\ {}^1/_2k]$$

To determine the critical gradient of temperature, which is the eigenvalue of our problem, it would be necessary to minimize expression (2.4) with respect to k with fixed remaining parameters and consider (2.4) as an implicit function of A and k

$$F(A,k)=0,\qquad \frac{\partial A}{\partial k}=-\frac{\partial F/\partial k}{\partial F/\partial A}$$

However this procedure, after the determination of $k_0(A)$ and its substitution into function $F(A,k)$, would result in a very cumbersome expression for A_*.

For this reason we minimize (2.4) approximately with respect to k by separate minimization of functions $\gamma_0(k)$ and $\gamma_1(k)$. This does not lead to any substantial error in the determination of A_*, since k_{01} and k_{02} which minimize $\gamma_0(k)$ and $\gamma_1(k)$ are close to each other, and functions $\gamma_0(k)$ and $\gamma_1(k)$ virtually coincide for $k=k_{01}$ and $k=k_{02}$ (**).

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*) To check the applicability of the Bubnov-Galerkin method to the Rayleigh problem, the critical Rayleigh number was calculated for an incompressible fluid, and the results were compared with those of the known exact solution. It was found that for the function $f_1^{(1)}$ in (2.1) $\gamma_0=1802$ (with the minimizing $k_0=3.1$); for the sum $\alpha_1f_1^{(1)}+\alpha_2f_2^{(1)}$, $\gamma_0=1712$ ($k_0=3.05$), and for function $f_1^{(2)}$ in (2.2) and the sum $\alpha_1f_1^{(2)}+\alpha_2f_2^{(2)}$, $\gamma_0=1707.8$ ($k_0=3.1$). All these results are very close to the exact solution $\gamma_0=1707.8$. However an unfortunate choice of the approximating function, e.g. $f_2^{(2)}$ in (2.2) yields $\gamma_0\approx 3\cdot10^5$ ($k_0=6$), differing considerably from the exact solution, i.e. such a velocity profile is not dangerous as regards the disruption of stability.

**) Thus for function $f_1^{(1)}$ in (2.1) we have: $k_{01}=3.1$, and the minimum of function γ_0 is $\gamma_0(k_{01})=1802$ with $\gamma_1(k_{01})=74.1$); $k_{02}=2.8$ and $\gamma_1(k_{02})=73.4$ (with $\gamma_0(k_{02})=1837$). The corresponding values for function $f_1^{(2)}$ in (2.2) are:

$$k_{01}=3.1,\quad \gamma_0(k_{01})=1707.8,\quad \gamma_1(k_{01})=70.5$$

$$k_{02}=2.8,\quad \gamma_0(k_{02})=1736,\quad \gamma_1(k_{02})=69.2$$

In the following we shall assume $k_0 = 3.1$, $\gamma_0(k_0) \approx 1700$ and $\gamma_1(k_0) \approx 70$. ($\xi/\nu = 1$ was assumed for simplicity).

It is convenient to write the solution of Eq. (2.4) for the dimensionless temperature gradient γ_* in the form:

$$\gamma_* = \gamma_0 (1 + l_{01}^4 + l_{01}^2 l_{21}^2)(1 + l_{31}^4 l_{20}^2)^{-1}, \qquad \gamma_* = \frac{\beta g l_0^4 A_*}{\nu\varkappa} \tag{2.5}$$

$$l_{01} = l_0/l_1, \; l_{21} = l_2/l_1, \; l_{31} = l_3/l_1 \quad \text{etc.}$$

where in addition to the numbers $\gamma_0 = 1700$ and $\gamma_1 = 70$ we have introduced three dimensions of length constructed from parameters of the problem

$$l_1 = \left[\frac{\gamma_0\nu\varkappa}{g^2(\partial\rho/\partial p)_T\,(1 - c_v/c_p)}\right]^{1/4}, \quad l_2 = \left[\frac{\gamma_1\nu\varkappa}{1 - c_v/c_p}\left(\frac{\partial\rho}{\partial p}\right)_T\right]^{1/2}, \quad l_3 = \left[\frac{\gamma_0\nu\varkappa}{g^2\,(\partial\rho/\partial p)_T}\right]^{1/4} \tag{2.6}$$

Obviously $l_1 \geqslant l_3$ with the equality sign applicable for $c_v/c_p \ll 1$.

Depending on the fluid parameters three fundamental cases may occur: (1) $l_2 < l_3 \leqslant \leqslant l_1$; (2) $l_3 \leqslant l_1 < l_2$, and, finally, (3) $l_3 < l_2 < l_1$ (for $(1 - c_v/c_p) \ll 1$).

For a given distance l_0 between the planes the criterion of convection onset will vary depending on the parameters of the fluid (cases 1–3) and, also, on the relationship between length l_0 and parameters l_1, l_2 and l_3.

We pass to numerical estimates, and will consider the cases of a "typical" fluid away from the critical point. Assuming

$$\left(\frac{\partial\rho}{\partial p}\right)_T = 10^{-10}\frac{\text{sec}^2}{\text{cm}^2}, \quad \nu = \varkappa = 10^{-3}\frac{\text{cm}^2}{\text{sec}}, \quad 1 - \frac{c_v}{c_p} = 10^{-3}, \quad g = 10^3\,\frac{\text{cm}}{\text{sec}^2} \tag{2.7}$$

we then obtain

$$l_2 = 2.5\cdot 10^{-6}\,\text{cm}, \; l_3 = 2\,\text{cm}, \; l_1 = 10\,\text{cm}$$

Let us consider the case of $l_2 < l_3 < l_1$ correspnding to (2.7). Since the analysis of the various possibilities resulting from (2.6) is quite simple, we shall adduce the final results only.

Six possible areas to be distinguished for the distance l_0 between planes. The criteria of convection onset are of the following form:

$$l_0 < l_2 l_{31}^2 \tag{2.8.1}$$

$$\gamma_* = \gamma_0 l_{13}^4 l_{02}^2 = \frac{\gamma_0 l_0^2}{\gamma_1\nu\varkappa\,(\partial\rho/\partial p)_T}$$

$$l_2 l_{31}^2 < l_0 < l_2 \tag{2.8.2}$$

$$\gamma_* = \gamma_0 (1 - l_{31}^4 l_{20}^2) = \gamma_0\left[1 - \gamma_1\nu\varkappa l_0^{-2}\left(\frac{\partial\rho}{\partial p}\right)_T\right]$$

$$l_2 < l_0 < l_3 \tag{2.8.3}$$

$$\gamma_* = \gamma_0(1 + l_{01}^4 - l_{31}^4 l_{20}^2) = \gamma_0 + \gamma_0 l_{01}^4 - \gamma_0\gamma_1\nu\varkappa l_0^{-2}(d\rho/dp)_T$$

$$l_3 < l_0 < l_1 \tag{2.8.4}$$

$$\gamma_* = \gamma_0(1 + l_{01}^4 + l_{01}^2 l_{21}^2) = \gamma_0 + \gamma_0 l_{01}^4 + \gamma_1 l_0^2 g^2 (d\rho/d\rho)_T^2$$

$$l_1 < l_0 < l_1 l_{12} \tag{2.8.5}$$

$$\gamma_* = \gamma_0 l_{01}^4 + \gamma_0$$

$$l_0 > l_1 l_{12} \tag{2.8.6}$$

$$\gamma_* = \gamma_0 l_{01}^4 + \gamma_0 l_{01}^2 l_{21}^2 = \gamma_0 l_{01}^4 + \gamma_1 l_0^2 g^2 (d\rho/dp)_T^2$$

The numerical estimates (2.7) show that for the "typical" fluid considered here of practical interest are areas $l_2 < l_0 < l_1$ and $l_1 < l_0$. In these cases (2.8.3–2.8.5) the critical gradient is equal to the sum of the Rayleigh temperature gradient γ_0 and of that of Schwarzschild $\gamma_0 \, l_{01}^4$. These are, obviously, limit cases with respect to parameter $l_{01}^4 \lesssim 1$. (Formulas (2.8.3, 2.8.4) contain, also, the first correction factors). We note that Sorokin [2] had derived on the basis of a qualitative analysis the criterion $l_{03}^4 \lesssim 1$ which coincides with the one obtained here for $(1 - c_v/c_p) \approx 1$ only.

We note that the estimates (2.7) of the problem parameters are "typical". For a highly viscous heat-conducting fluid parameter l_3 may increase to such an extent that it becomes expedient to consider area (2) where corrections to the Rayleigh criterion are proportional to l_0^{-2}, and generally speaking, also, area (1) in which appears a new criterion of convection onset – the critical gradient of temperature which is no longer proportional to l_0^{-4}, as in the Rayleigh problem for an incompressible fluid, but to l_0^{-2}. The corrections to the Schwarzschild criterion for area (6) are of a distinctive kind.

The more "exotic" cases of $l_3 \leqslant l_1 < l_2$ and $l_3 < l_2 < l_1$ in relationship (2.5) can be investigated in the same manner.

The application of the derived formulas in the analysis of singularities of convection in proximity to the critical point will be given in separate paper.

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BIBLIOGRAPHY

1. Landau, L. D. and Lifshits, E. M., Mechanics of Continuous Media, 2nd ed., M., Gostekhizdat, 1953.

2. Sorokin, V. S., On the stability of a nonuniformly heated gas in a gravitational field. PMM Vol.17, №2, 1953.

3. Filippenko, L. G., The condition of absence of convection in an unbounded real fluid. Izv. Akad. Nauk SSSR, MZhG, №4, 1967.

4. Spiegel, E. A. and Veronis, G., On the Boussinesq approximation for a compressible fluid. Astrophys. J., Vol.131, №2, 1960.

5. Jeffreys, H., The instability of a compressible fluid heated below. Proc. Cambridge Philos. Soc., Vol.26, 1930.

6. Sorokin, V. S., Variational method in the theory of convection. PMM Vol.17, №1, 1953.

7. Zhukhovitskii, E. M., Application of the Galerkin method to the problem on the stability of nonuniformly heated liquid. PMM Vol.18, №2, 1954.

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